Math 313 – *Classwork 17: Interpreting the Dual LP*

1. A farmer has 500 acres of land and wishes to determine the acreage allocated to the following three crops: wheat, corn and soybeans. The man-days, preparation cost, and profit **per acre** of the three crops are summarized in the table below:

Crop	Man-Days/acre	Preparation Costs ($/acre)	Profit ($/acre)
Wheat	6	100	60
Corn	8	150	100
Soybeans	10	120	80
Available	5,000 Man-Days	$60,000	

The primal linear program is:

$$\begin{aligned} \text{Max.} \quad & 60x_1 + 100x_2 + 80x_3 \\ \text{S.t.} \quad & \\ & 6x_1 + 8x_2 + 10x_3 \leq 5,000 \\ & 100x_1 + 150x_2 + 120x_3 \leq 60,000 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) For this primal linear program,

i. define the variables and be clear what units are used for each variable;

ii. determine the units for each coefficient in the program; and

iii. label the constraints so that it is clear what they represent.

iv. Explain what goal leads to this linear program.

The dual linear program is:

$$\begin{aligned} \text{Min.} \quad & 5,000y_1 + 60,000y_2 \\ \text{S.t.} \quad & \\ & 6y_1 + 100y_2 \geq 60 \\ & 8y_1 + 150y_2 \geq 100 \\ & 10y_1 + 120y_2 \geq 80 \\ & y_1, y_2 \geq 0. \end{aligned}$$

(b) For the dual linear program:

i. using the objective function, determine the units for each of the dual variables;

ii. check that the units for the left-hand side and right-hand side of each of the constraints are the same;

iii. explain what the first dual constraint represents and give the constraint a name; and

iv. explain what the second dual constraint represents and give the constraint a name.

(c) Identify a point of view which leads to the dual linear program.

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2. A steel company has to ship the ore needed to make steel from the three different mines they own to the four different plants where the steel will be made. The amount of ore (in tons) which is available at the different mines, the minimum amount of ore (in tons) needed at the plants to process current orders, and the shipping cost per ton from the various mines to the various plants is given in the following table:

	Shipping Costs ($/ton)			Available at Mine
	Plant 1	Plant 2	Plant 3	(tons)
Mine 1	400	700	850	1200
Mine 2	300	600	700	1800
Plant Need (tons)	900	1100	800	

How must the shipments be made so that the demands are met at a minimal shipping cost to the company?

The primal linear program is:

$$
\begin{array}{lrcrcrcrcl}
\text{Min.} & 400x_1 & + & 700x_2 & + & 850x_3 & + & 300x_4 & + & 600x_5 + 700x_6 & \\
\text{S.t.} & 1x_1 & & & & & + & 1x_4 & & & \geq\ 900 \\
& & & 1x_2 & & & & & + & 1x_5 & \geq 1100 \\
& & & & & 1x_3 & & & & + \quad 1x_6 & \geq\ 800 \\
& 1x_1 & + & 1x_2 & + & 1x_3 & & & & & \leq 1200 \\
& & & & & & & 1x_4 & + & 1x_5 + \quad 1x_6 & \leq 1800 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0. & & & & & & & & &
\end{array}
$$

(a) For this primal linear program,

i. define the variables and be clear what units are used for each variable;

ii. determine the units for each coefficient in the program; and

iii. label the constraints so that it is clear what they represent.

iv. Explain what goal leads to this linear program.

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The dual linear program is:

$$
\begin{array}{lrrrrrrl}
\text{Max.} & 900y_1 & + 1100y_2 & + 800y_3 & + 1200y_4 & + 1800y_5 & & \\
\text{S.t.} & & & & & & & \\
 & 1y_1 + & & & + & 1y_4 & & \geq 400 \\
 & & 1y_2 & & + & 1y_4 & & \geq 700 \\
 & & & 1y_3 + & & 1y_4 & & \geq 850 \\
 & 1y_1 + & & & & & + & 1y_5 \geq 300 \\
 & & 1y_2 & & & & + & 1y_5 \geq 600 \\
 & & & 1y_3 & & & + & 1y_5 \geq 700 \\
 & y_1, y_2, y_3 \geq 0, & & y_4, y_5 \leq 0. & & & &
\end{array}
$$

(b) For the dual linear program:

i. using the objective function, determine the units for each of the dual variables;

ii. check that the units for the left-hand side and right-hand side of each of the constraints are the same;

iii. explain what the first dual constraint represents and give the constraint a name; and

iv. explain what the second dual constraint represents and give the constraint a name.

(c) Identify a point of view which leads to the dual linear program.

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(a) (i) The variables are x_1, x_2 and x_3.
The units are units of weight like kg, gram per acre.

(ii) The units for each coefficient is $ (dollar) i.e unit of cost/profit.

(iii) The first constraint, $6x_1 + 8x_2 + 10x_3 \leq 5000$ gives the total man-days/acre to cultivate the crops.
The second constraint, $100x_1 + 150x_2 + 120x_3 \leq 60000$. is the preparation cost per acre of the three crops.

(iv) The goal of this linear program is to maximize the total profit subject to the given two constraints and determine x_1, x_2, x_3.

(b) (i) The units of the dual variables y_1 and y_2 are total mandays/acre and total preparation cost/acre respectively.

(ii) The units of the constraints are $ (dollar) written is same in both LHS and RHS.

(iii) The first dual constraint $6y_1 + 100y_2 \geq 60$. The total profit to cultivate wheat and the second constraint represents the total profit to cultivate corn in the land.

(c) The dual problem leads to minimize the total costs subject to the constraint and for this determine y_1, y_2.

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(a)i. Notice that the coefficients of the variables are the per ton shipping costs between the various mines and plants. So x_1 is the number of tons of ore shipped from mine 1 to plant 1, x_2 is the number of tons of ore shipped from mine 1 to plant 2, x_3 is the number of tons of ore shipped from mine 1 to plant 3. Similarly, x_4 is the number of tons of ore shipped from mine 2 to plant 1, x_5 is the number of tons of ore shipped from mine 2 to plant 2 and x_6 is the number of tons of ore shipped from mine 2 to plant 3.
ii. The coefficients of the variables in the objective function are the per ton shipping costs so their units are \$/ton. The constrant RHS values are the total avilability and required demands at each of the mines and plants so their units are in tons, and the 1 in the coefficient of all the constraints is simply tons.
iii. The first constraint sets up the limit that the total units reaching plant 1(i.e. x_1 from mine 1 and x_4 from mine 2) must exceed their demand, 900. Similarly the second constraint is for the demand of plant 2 and the third constraint is for the demand at plant 3. Then, the fourth and fifth constraints are for the total supply at each mine, i.e. it takes the sum of the ore leaving each mine and sets it less than or equal to the total availability, hence the fourth constraint is for mine 1 supply and the fifth constraint is for mine 2 supply.
iv. For this linear program, we are trying to minimize the total shipping costs while ensuring that the demand at each plant is fulfilled while not exceeding the supply at each mine.

(b)i. The objective function of the primal program was to minimize the total cost. The dual program has the supplies and demands by tonnage as the coefficients of the variables, and the objective function of the primal and dual would have to be the same, so it has to equate to cost. So the dual variables have units of \$/ton
ii. The 1 in the coefficient of the dual variables in the constraints has no unit and the variables are in \$/ton, so the LHS is in \$/ton, and the RHS values are the shipping costs per ton, so the RHS values are in \$/ton as well. Hence they are the same unit.
iii. The first dual constraint is $y_1 + y_4 \leq 400$. This tells us that the total cost per ton of ore shipped between mine 1 and plant 1 cannot exceed \$400/ton.
iv. The second dual constraint is $y_2 + y_4 \leq 700$ so this tells us that the total cost per ton of ore shipped between mine 1 and plant 2 cannot exceed \$700/ton.

(c) The primal problem was seeking to minimize the total cost while satisfying demand and not exceeding supply. So, the dual problem can be thought as maximizing the utilization of resources at each plant-mine combination while making sure the per ton shipping costs do not exceed the given values.

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